

# WJEC (Wales) Physics A-level

## Topic 4.1: Capacitance Notes

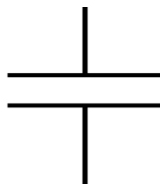
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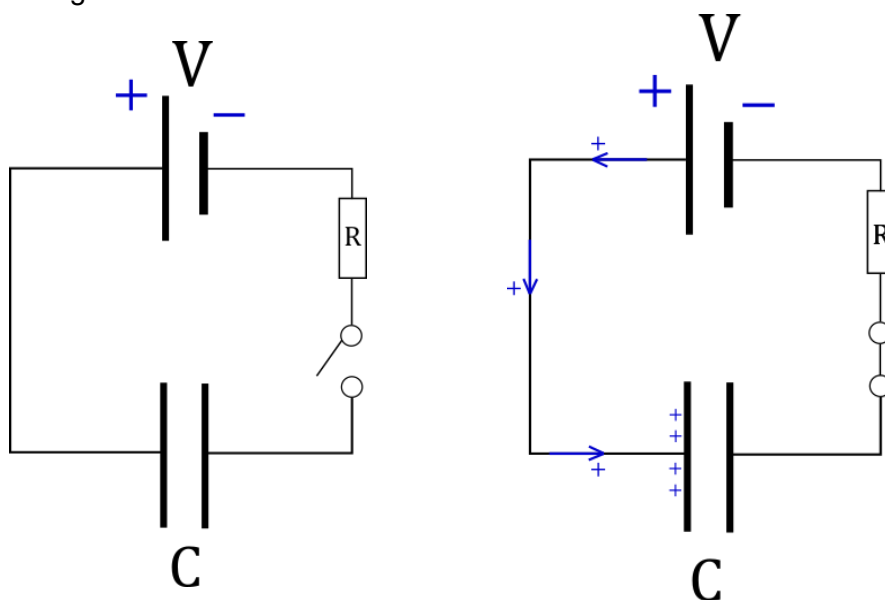


## Capacitors

A capacitor is a type of electrical device which is used to **store charge**. Capacitors consist of two **parallel conducting plates** separated by a small distance. The circuit symbol for a capacitor is shown here:



To charge capacitors a potential difference needs to be applied across them. To see why, let's imagine connecting a cell across a capacitor as shown below in the left-hand diagram:



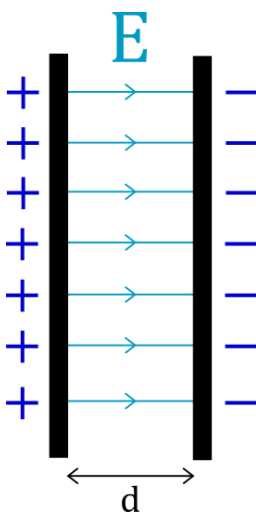
Here we have connected a cell providing a potential  $V$ , a resistor  $R$  and a switch. We have added a resistor just so that the current flowing in the circuit can be controlled by changing the value of  $R$ . In this diagram, the switch is open meaning that no current can flow in the circuit.

Now let's think about what happens when the switch is closed. Seeing as the circuit becomes complete, current starts to flow due to the cell. Positive charges begin to flow from the positive terminal of the battery, these reach the capacitor but can travel no further due to the gap between the plates. **This causes positive charges to accumulate on the left-hand capacitor plate.** This is the situation shown on the right-hand diagram.



Whilst this process is happening, the **opposite process** is happening from the **negative terminal of the cell**. Positive charges are drawn towards the negative terminal, this effect pulls positive charges from the other capacitor plate, leaving only negative charges behind, leading to an accumulation of **negative charges** on this plate. **The plates always have an equal and opposite amount of charge.**

As positive charges build up on the left-hand plate, and negative charges on the right, an **electric field is generated in the space between the plates**. Electric field lines point away from positive charges, and in between the plates the strength of the field is constant. Below is an illustration of the electric field between the plates of the capacitor, where the field has strength  $E$ .



**This electric field creates a potential difference between the plates.** To find the relation between the field and the potential imagine placing a small charge  $q$  on the inner side of the left plate, then moving it across to the right-hand plate. The electric field will exert a force  $F = qE$  on the particle, and using that

$Work = Force \times Distance$ , we find the work done on the particle is  $W = qEd$ , using the fact that the distance between the plates is  $d$ . The potential difference between two points is defined as the change in potential energy *per unit charge*, so dividing  $W$  by  $q$  gives:

$$V = Ed$$

This tells us that as the charges accumulate on each plate, the increasing electric field causes an increased potential difference between the plates. **The plates will continue to charge until the potential difference across them is equal to the potential difference supplied by the cell.**





## Capacitance

Seeing as capacitors store charge, **the capacitance is a way of measuring how good a capacitor is at storing charge.**

If we apply a potential difference  $V$  across a capacitor causing a charge  $Q$  to accumulate on the capacitor, then the capacitance is given by:

$$C = \frac{Q}{V}$$

This is the charge stored per unit potential difference, meaning it has units  $CV^{-1}$ . **This unit is given the name Farad, with symbol  $F$ .** This means a 1 Farad capacitor will store 1 Coulomb of charge per unit of potential difference applied across it.

An important point is that the  $Q$  in the above equation represents the charge stored on the **positive plate**. On the negative plate a charge  $-Q$  will be stored, **meaning the net charge on a capacitor is always zero.**

## Energy stored in a capacitor

Not only do capacitors store charge, they also store energy. Thinking back to the example of the capacitor being charged by a cell, initially the potential difference between the cell and capacitor is  $V$  because the potential across the capacitor starts at zero. The first charge  $q$  moved to the capacitor by the cell will move through this potential  $V$ , meaning the charge receives an energy  $qV$ .

This single charge  $q$  on the capacitor causes a **small increase** in the potential across the capacitor, which causes an **equally small drop in potential between the cell and capacitor**. This means the next charge to move from the cell to the capacitor will move through a **potential which is slightly less than  $V$** . This means as the capacitor charges, the potential the charges are moving through constantly decreases until it reaches zero. So on average, the charges move through a potential  $\frac{V}{2}$  during the charging process. If the cell transfers a total charge  $Q$  to the capacitor, then the total energy stored on the capacitor must be:

We can express the stored energy  $U$  in terms of the **capacitance** by substituting  $Q = CV$  into the above equation. This gives two more equations for  $U$ :

$$U = \frac{QV}{2}$$





$$U = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

## What Makes a Good Capacitor?

The capacitance is the **charge stored per unit potential difference**, this means that any factor which **increases the charge stored** on the capacitor or **decreases the potential difference** across it will increase the capacitance.

For a parallel plate capacitor, **increasing the surface area of the plates will allow the device to store more charge**. Also remembering that  $V = Ed$ , decreasing the distance between the plates  $d$  will reduce the potential difference between the plates, leading to a larger capacitance.

These two points imply that the capacitance must be proportional to  $\frac{A}{d}$  where  $A$  is the surface area of one of the parallel plates. The constant of proportionality is  $\epsilon_0$ , **called the permittivity of free space, which has a value  $\epsilon_0 = 8.85 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$** .

Putting these together gives the **capacitance of a parallel plate device as:**

$$C = \frac{\epsilon_0 A}{d}$$

Capacitors usually don't have a vacuum separating their plates. This is because the value of a device's capacitance can be **increased by inserting a type of material called a dielectric**. It is generally **easier to generate an electric field inside a dielectric**, meaning that fewer charges are needed to create the same potential difference across the plates.

**Dielectric materials are insulators**, meaning that they do not allow charge to flow across the plates. If they did allow charges to flow then the device would no longer act as a capacitor, so conducting materials couldn't be used between the plates.

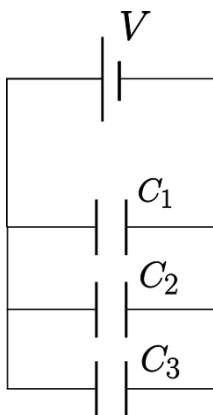




## Combining Capacitors

Similarly to how resistors can be combined to give a section of a circuit with some overall resistance, capacitors can be combined to produce different capacitances.

**In parallel:** Consider the following arrangement of capacitors arranged in parallel:



The potential difference across each capacitor is  $V$ , meaning that the **total charge** stored by this arrangement is:

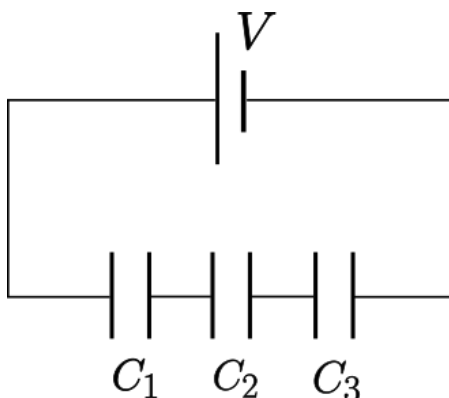
Using the usual definition of capacitance gives the capacitance of this arrangement as:

$$\begin{aligned} Q_{tot} &= C_1V + C_2V + C_3V \\ \frac{Q_{tot}}{V} &= C_{tot} = C_1 + C_2 + C_3 \end{aligned}$$

**Therefore capacitances add in parallel.**

**In series:**

Now let's arrange the capacitors in series with the same potential across them:





If the potential across each capacitor is  $V_i$  where  $i$  represents the  $i$ 'th capacitor, then:

$$V = V_1 + V_2 + V_3$$

The capacitors must all hold the **same charge** seeing as the charge lost from one plate must have been gained by the other in the charging process. Dividing by this charge gives:

$$\frac{V}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} + \frac{V_3}{Q}$$

The term on the left is the inverse of the total capacitance, and each term on the right is the inverse of the individual capacitances, meaning:

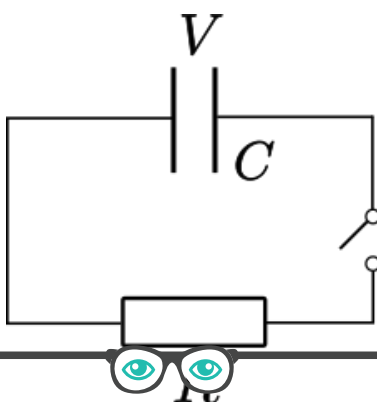
$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

**Meaning capacitances in series add inversely.** This shows capacitance adds in the opposite way to how resistance adds.

## Charging and Discharging Capacitors

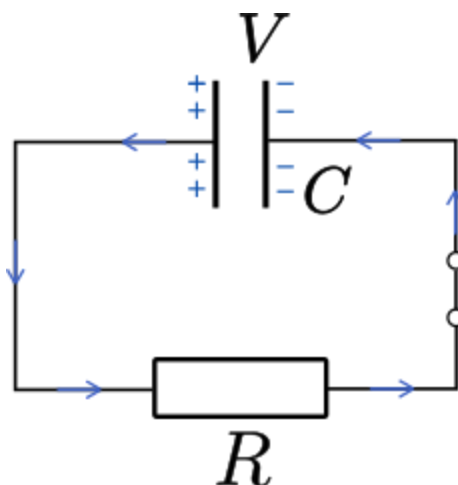
Capacitors are useful not only because they can be charged, but also because they can be **discharged**. A capacitor can be used to provide a source of current to a circuit similar to how a battery does, with the important difference that a **capacitor cannot provide a constant current (or emf)**.

To investigate why this is the case, let's imagine taking a charged capacitor holding a charge  $Q$  at a potential  $V$ , and placing it into a circuit with a resistor  $R$ :





Now let's consider what happens when the switch is closed. When this happens, a current immediately begins to flow in the circuit caused by the potential difference across the capacitor:



The blue arrows indicate the direction of the current flow. The potential difference across the resistor is always the same as the potential across the capacitor, and is given by  $V = IR$ . This means the **current is proportional to the potential difference across the capacitor**. The current causes the **charge on the capacitor to decrease**, and because  $Q = CV$ , the potential difference across the capacitor decreases.

These factors mean that the **rate at which charge is being lost** from the capacitor is proportional to the **amount of charge left on the capacitor**. This means that the **charge decays exponentially** during the discharge, given by:

$$Q = Q_0 e^{-\frac{t}{RC}}$$

$Q_0$  is the initial charge on the capacitor. The factor  $RC$  has units of time and is called the **time constant**. Larger time constants cause slower discharges. Seeing as the potential difference and current are both proportional to the charge left on the capacitor, both quantities decay in a similar way:

$$V = V_0 e^{-\frac{t}{RC}} \quad I = I_0 e^{-\frac{t}{RC}}$$





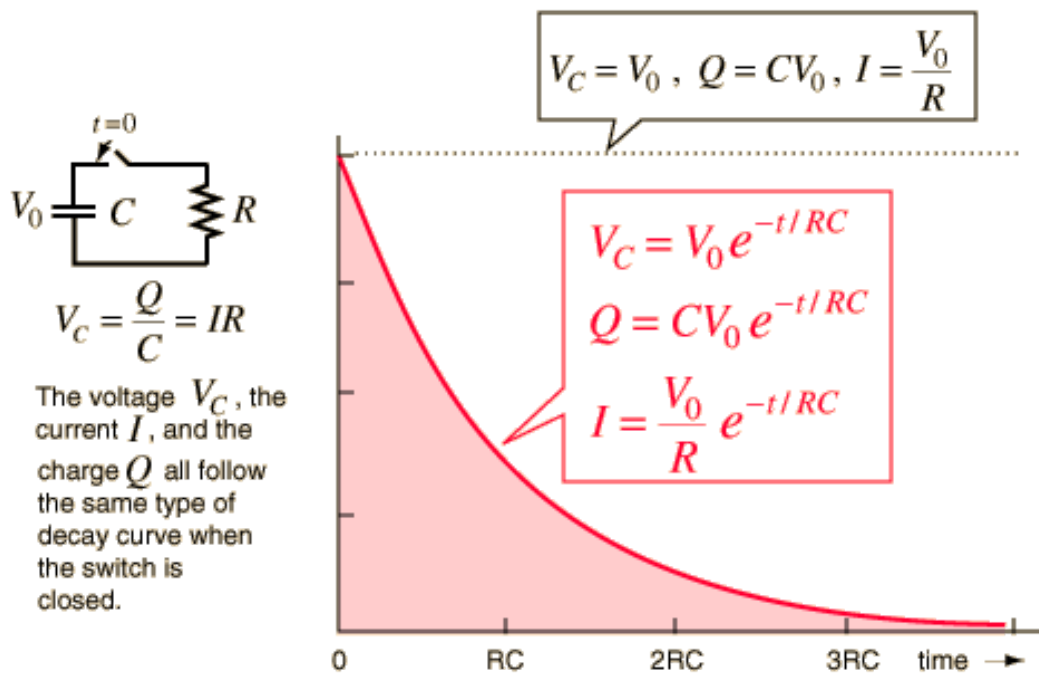
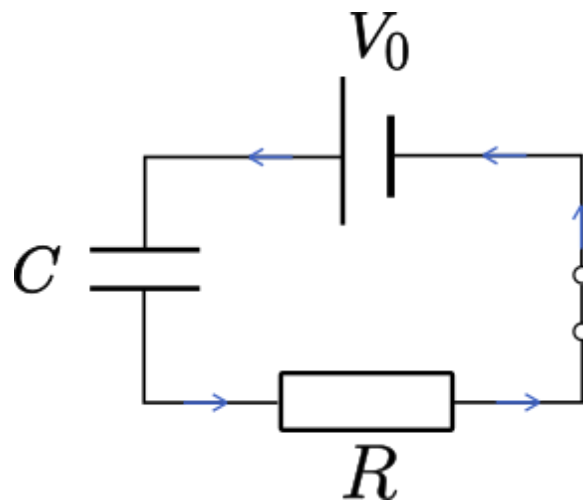


Image source: <http://hyperphysics.phy-astr.gsu.edu/hbase/electric/capdis.html>

This graph displays the shape of these curves. The time axes shows intervals in terms of the time constant.

These equations describe a capacitor **discharging**. Let's consider the circuit we started out with to investigate how the potential, current and charge vary during **charging**:



Initially there is no charge on the capacitor so the potential across it must be zero. After a long enough time, the potential across the capacitor must reach  $V_0$ . This implies the potential across the capacitor **during charging** takes the form:

$$V = V_0 \left( 1 - e^{-\frac{t}{RC}} \right)$$

Using the fact that the potential and charge are proportional, we get a similar equation for the charge:

$$Q = Q_0 \left( 1 - e^{-\frac{t}{RC}} \right)$$

This situation is not the same for current however, seeing as eventually the charge on the capacitor will reach a constant value of  $Q_0$ , meaning that there is **no charge flowing in the circuit**. Hence the **current still decays to zero** in the same way even for charging, following the equation (where  $I_0$  is the initial current in the circuit)

$$I = I_0 e^{-\frac{t}{RC}}$$

